UWEC Math meet: Some sample problems

Simplify final answers and place them in the space provided.

1. A vending machine contains dimes, nickels, and quarters. There are 4 more dimes than quarters and 5 more nickels than dimes. Let x be the number of quarters. Write down an expression in x for the total value, in cents, of all of the coins in the machine. Make sure to simplify.

Answer:

2. How many (x, y) solutions are there to

$$\frac{x+\frac{1}{y}}{y+\frac{1}{x}} = 2025, \ x+y \le 10,000$$

with x and y positive integers?

Answer: _____

3. The sum of six consecutive odd numbers agrees with the sum of five consecutive even numbers. If the smallest odd is 5 then what is the largest even?

Answer:

4. You have 100 meters of fencing to build a circular enclosure. How much area will the enclosure contain?

Answer: _____

5. You shine a light on a coin from a distance of 20 cm. On a wall 10 meters from the light source its shadow is a circle of diameter 2 meters. What is the diameter of the coin in cm?

Answer: _____

6. Draw two adjacent circles, one of radius 2 feet, another of radius 4 feet. Draw two lines tangent to each of these circles. What is the shortest distance from the point where these lines meet to the edge of the smaller circle?



Answer: _____

7. If $2^x = 3$, then what is 4^{-x} ?

Answer: _____

8. Let
$$f(x) = \sin\left(\frac{\pi x}{2}\right)$$
. Evaluate $f(2025)$.

Answer: _____

9. Two runners start at the same point and run along a 1 mile loop. They run in opposite directions and one of the runners is twice as fast as the other. After the faster runner has finished seven loops, how many times have the runners met? (Include the moment when they first start running.)

Answer	;
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10. Two cake-couriers must carry a cake first 2 miles up a hill then 2 miles down, then along a 2 mile flat straightaway. Their downhill pace is three times their uphill pace and twice their pace over flat ground. It takes them 40 minutes to make it up and then down the hill. What is their total travel time? (In minutes)

Answer: _____

11. A survey reports that precisely 33.75% of the sampled population answered a particular question with 'yes.' What is the smallest number of people that could possibly have answered the survey?

Answer:

12. Suppose you have a $3 \times 3 \times 3$ inch cube made up of 1 inch cubes. You paint the outside of the large cube. If you then break the cube into its 1 inch pieces and choose one of these pieces at random, then what is the probability that you will pick a cube painted on exactly two sides?

13. Start with an equilateral triangle of area 1. Add an equilateral triangle of area $\frac{1}{9}$ to the middle of every side of this triangle. Now add to the middle of every side of the resulting object an equilateral triangle of area $(\frac{1}{9})^2$. Iterate this addition 100 times. What is the area of the resulting object? Round your answer to the nearest thousandth.



Answer: _____

14. A sequence begins with 2025 as its first term. Each subsequent term is the sum of the squares of the digits of the preceding number. What is the 2025th term in this sequence?

Answer: _____

15. Below you see a circle of radius 1 and a 2×1 rectangle both centered at (0,0). What is the area of their intersection? Round your answer to the thousandth's place.



Answer:

16. In the diagram below, ABCD is a square whose sides are each 4 units long. The length of the shortest path from A to C following the lines of the diagram is 8 units. How many different paths of length 8 are there from A to C?





17. Let
$$f(x) = \frac{\ln (x+1)}{4}$$
. What value of x results in $f(f(f(x))) = 0$?

Answer:

18. After waking up and not having his coffee, Chris stumbles one step to the right, then two to the left, then three to the right and so on. After completing his stumble 2025 steps to the right, how far to the right or left is Chris from where he began?